Pilot Structure Design to Increase Wireless Channel Capacity for High-Speed Railway

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***Abstract*—In this paper, we design a pilot structure in an orthogonal frequency division multiplexing (OFDM) system for high-speed railway (HSR). Ergodic capacity is derived and its lower bound is obtained in closed form. Two schemes of pilot placement in time and frequency domains are proposed which maximizes the lower bound of the ergodic capacity. The ergodic capacities of the two proposed schemes are obtained by computer simulation and compared. It is shown that the proposed schemes achieve higher ergodic capacity than the random pilot placement scheme.**

***Index terms* — Orthogonal frequency division multiplexing, high-speed railway, channel estimation, pilot structure, ergodic capacity.**

I. INTRODUCTION

Wireless communication for high-speed railway (HSR) has received increasing attentions recently as the train speed increases [1]-[3]. Due to high mobility of train, wireless channel shows fast fading [4]. Although the channel state information of a fast fading channel can be estimated by using pilots [5], [6], its estimation error is not negligible and it significantly degrades the performance of the communication [7]-[9].

Thus, most of previous works on wireless communication for HSR focused on enhancing the pilot-assisted channel estimation [10]-[12]. In [10], an optimal pilot insertion interval in a time domain is obtained to maximize user throughput. In [11], an optimum pilot percentage that can maximize the spectral efficiency is determined and in [12], the ratio of energy allocated between data symbols and pilot symbols is also optimized. However, in a multi-carrier system such as orthogonal frequency division multiplexing (OFDM), pilot structure design in time and frequency domains has not been studied yet.

In this paper, we design a pilot structure of an OFDM system for HSR which has a transmitter on a train and a receiver on a railroad side. We derive an ergodic capacity and obtain its lower bound by using Jensen's inequality. We formulate a pilot structure design problem which maximize the lower bound of the ergodic capacity. To find the solution of this problem, we propose two pilot placement schemes. The performances of the proposed pilot placement schemes are shown by computer simulation.

The rest of this paper is organized as follows. We describe a system model in Section II. In Section III, a pilot structure design problem is formulated and two pilot placement schemes are proposed to obtain a solution of this problem. Simulation results are shown in Section IV and conclusions are drawn in Section V.

II. SYSTEM MODEL

Consider an OFDM system for HSR which consists of a transmitter on a train and a receiver on railroad side. Assume that the transmitter and the receiver are equipped with a single antenna. Suppose that the total spectrum is divided into N subchannels, each with bandwidth W. Suppose that a frame is divided into M time slots, each with duration T.

Let $h_n(m)$ denote the coefficient of the channel between the transmitter and the receiver at mth time slot on nth subchannel, $m = 1, \cdots, M$, $n = 1, \cdots, N$, which is modeled as a Rayleigh fading channel. Assume that the coefficient of a subchannel remains constant during each time slot, and is correlated in time and frequency domains. Jake's model is used for the autocorrelation function of the channel, which is given by [13]

$$\begin{aligned} R_{n_1,n_2}(m2-m1) &= E\left[h_{n_1}(m_1)h^*_{n_2}(m_2)\right] \\ &= J_0(2\pi f_d(m_2-m_1)T)\frac{1-j2\pi(n_2-n_1)\sigma_t/T}{1+4\pi^2(n_2-n_1)^2\sigma_t^2/T^2} \quad (1) \end{aligned}$$

where f_d is the maximum Doppler frequency, σ_t is the maximum delay spread, and $J_0(\cdot)$ is the zero-order Bessel function of the first kind.

For simplicity, mth time slot on nth subchannel is called as (m,n)th block. All blocks are divided into two types: reference blocks (RBs) and information blocks (IBs). In a RB, a pilot is transmitted to estimate its channel state information (CSI). Assume that the CSI of a RB is perfectly estimated by the pilot.

In an IB, an information symbol is transmitted. Let $x_n(m)$ denote an information symbol in (m,n)th IB. The received signal in (m,n)th IB is given by

$$y_n(m) = h_n(m)x_n(m) + n_n(m) \tag{2}$$

where $n_n(m)$ is an additive white Gaussian noise (AWGN) with zero mean and variance N_0.

An IB selects a RB which has highest correlation with it among RBs in the past and various subchannels, so that it adopts the CSI of the RB as its CSI. Let the estimated CSI used for (m,n)th IB is denoted by $\hat{h}_n(m)$ and the correlation between $h_n(m)$ and $\hat{h}_n(m)$ is denoted by $\rho_n(m)$. The channel coefficient on (m,n)th IB is given by

$$h_n(m) = \hat{h}_n(m) + e_n(m) \tag{3}$$

where $e_n(m)$ is the channel estimation error which is modeled as a complex Gaussian random variable with zero mean and variance $1-\rho_n(m)^2$ [14].

From (2) and (3), the received signal-to-noise ratio (SNR) on (m,n)th IB at the destination is given by

$$\begin{aligned}\gamma_n(m) &= \frac{P\left|\hat{h}_n(m)\right|^2}{P|e_n(m)|^2 + N_0} \\ &= \frac{\gamma\left|\hat{h}_n(m)\right|^2}{\gamma|e_n(m)|^2 + 1}\end{aligned} \tag{4}$$

where P is the transmit power at the transmitter and $\gamma = P/N_0$. The ergodic capacity of the (m,n)th IB is given by

$$\begin{aligned}C_n(m) &= E\left[\log_2\left(1+\gamma_n(m)\right)\right] \\ &= E\left[\log_2\left(1 + \frac{\gamma\left|\hat{h}_n(m)\right|^2}{\gamma|e_n(m)|^2+1}\right)\right].\end{aligned} \tag{5}$$

It is impossible to calculate this expectation exactly because the random variable $|e_n(m)|^2$ is in denominator. Using Jensen's inequality, $C_n(m)$ is lower bounded by

$$\begin{aligned}C_n(m) &\geq \frac{1}{\ln 2}\exp\left(A_n(m)\right)\Gamma\left(0, A_n(m)\right) \\ &\triangleq \tilde{C}_n(m)\end{aligned} \tag{6}$$

where $\Gamma(s,x)$ is the incomplete Gamma function, i.e.,

$$\Gamma(s,x) = \int_x^\infty t^{s-1}e^{-t}dt \tag{7}$$

and

$$A_n(m) = \frac{\gamma(1-\rho_n(m)^2)+1}{\gamma\rho_n(m)^2}. \tag{8}$$

The details of the derivation of (6) are in Appendix.

The indicator of the (m,n)th block, $m=1,\cdots,M$, $n=1,\cdots,N$ is defined as

$$\xi_{mn} \triangleq \begin{cases} 1, & \text{if } (m,n)\text{th block is an IB.} \\ 0, & \text{if } (m,n)\text{th block is a RB,}\end{cases} \tag{9}$$

The ergodic capacity of a frame can be obtained as the normalized sum of the ergodic capacity on all IBs. From (5) and (9), the ergodic capacity of a frame is given by

$$\begin{aligned}C &= \frac{1}{MN}\sum_{m,n}\xi_{mn}C_n(m) \\ &= \frac{1}{MN}\sum_{m,n}\xi_{mn}E\left[\log_2\left(1+\frac{\gamma\left|\hat{h}_n(m)\right|^2}{\gamma|e_n(m)|^2+1}\right)\right].\end{aligned} \tag{10}$$

From (6) and (10), the ergodic capacity of a frame is lower bounded by

$$\begin{aligned}C &\geq \frac{1}{MN}\sum_{m,n}\xi_{mn}\tilde{C}_n(m) \\ &= \frac{1}{\ln 2}\frac{1}{MN}\sum_{m,n}\xi_{mn}\exp\left(A_n(m)\right)\Gamma\left(0,A_n(m)\right) \\ &\triangleq \tilde{C}.\end{aligned} \tag{11}$$

III. Pilot Structure Design

In this section, we design a pilot structure which maximizes the lower bound of the ergodic capacity. As the number of RBs increases, the received SNR of IBs increases because its channel estimation accuracy increases, while the number of IBs decreases. Furthermore, the locations of RBs in time and frequency domain affect channel estimation accuracy. Thus, pilot structure design in both the number of RBs and their locations need to be considered to maximize the lower bound of the ergodic capacity.

A pilot structure design problem is formulated as

$$\max_{\xi_{mn}} \ \tilde{C} = \frac{1}{MN}\sum_{m,n}\xi_{mn}\tilde{C}_n(m) \tag{12}$$

subject to

$$\xi_{mn} \in \{0,1\}, \quad m=1,\cdots,M, \quad n=1,\cdots,N. \tag{13}$$

Constraint (13) indicates that the indicator ξ_{mn} can only have the binary value, i.e., zero or one.

A. Greedy Pilot Placement

Since the problem (12) is non-convex and the number of possible solutions is 2^{MN}, it is intractable to obtain an optimal solution analytically. We propose a greedy pilot placement (GPP) scheme which iteratively selects additional RB which maximizes the lower bound of the ergodic capacity. GPP scheme is based on a greedy algorithm, as shown in Algorithm 1.

The details of the Algorithm 1 are as follows. Initially, set indicators $\xi_{mn} = 1$, $m = 1, \cdots, M$, $n = 1, \cdots, N$, and $\xi_{11} = 0$. With these, it computes initial value of the lower bound of ergodic capacity $\tilde{C}_{old}$. Then, it computes the new lower bound of the ergodic capacity, $\tilde{C}_{new}(m,n)$, with $\xi_{mn} = 0$, $m = 1, \cdots, M$, $n = 1, \cdots, N$. A pair (m^*, n^*) is selected, such that $\tilde{C}_{new}(m^*, n^*)$ is maximum among all $\tilde{C}_{new}(m,n)$. If $\tilde{C}_{new}(m^*, n^*)$ is larger than $\tilde{C}_{old}$, $\tilde{C}_{old}$ is updated by $\tilde{C}_{new}(m^*, n^*)$ and set $\xi_{m^*n^*} = 0$. If $\tilde{C}_{new}(m^*, n^*)$ is smaller than $\tilde{C}_{old}$, the algorithm terminates.

Algorithm 1 Greedy Algorithm in GPP Scheme

Initialization
  Set $\xi_{mn} = 1$, $m = 1, \cdots, M$, $n = 1, \cdots, N$.
  Set $\xi_{11} = 0$.
  Set allocatepilot = true.
  Calculate $\tilde{C}_{old}$, using (11).
repeat
  $\tilde{C}_{new}(m,n)$:= Lower bound of the ergodic capacity calculated using (11), when $\xi_{mn} = 0$;
  $(m^*, n^*) = \arg\max_{m,n} \tilde{C}_{new}(m,n)$.
  if $\tilde{C}_{new}(m^*, n^*) \geq \tilde{C}_{old}$ **then**
    Set allocatepilot = true.
    Update $\tilde{C}_{old} = \tilde{C}_{new}(m^*, n^*)$.
    Set $\xi_{m^*n^*} = 0$.
  else
    Set allocatepilot = false.
  end if
until allocatepilot = true

B. Periodic Pilot Placement

The computational complexity of Algorithm 1 is $\mathcal{O}(M^2N^2)$. When the number of subchannels, N, or the number of time slots, M, is large, a computational complexity of Algorithm 1 is still too high to obtain its solution. To reduce the computational complexity, we propose a new pilot placement scheme called periodic pilot placement (PPP) scheme. In PPP scheme, the RBs are periodically placed in the time and frequency domains to simplify procedure of the pilot placement. Let d_t and d_f denote the interval of the RB in the time and frequency domains, respectively.

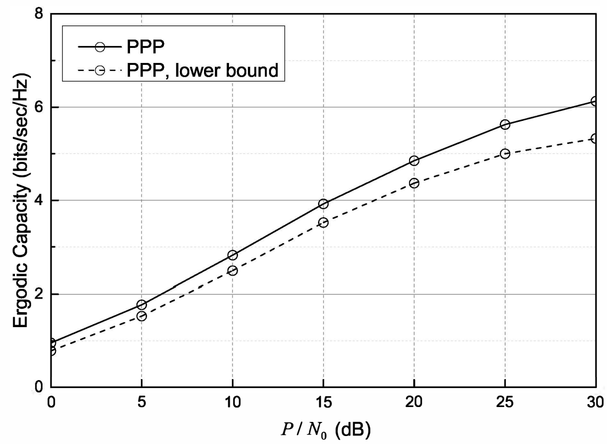

Fig. 1. Ergodic capacity for the proposed PPP scheme.

Since the number of time slots is M and the number of subchannels is N, the maximum possible values of d_t and d_f are M and N, respectively. If the d_t and d_f are selected, the indicator of (m,n)th block is given by

$$\xi_{mn} = \begin{cases} 0, & \text{if } \bmod(m, d_t) = 1 \text{ and } \bmod(n, d_f) = 1, \\ 1, & \text{otherwise}, \end{cases} \tag{14}$$

for all m and n, where $\bmod(\cdot,\cdot)$ stands for modulo operation.

Now the lower bound in (12) is maximized over d_t and d_f instead of ξ_{mn}, that is,

$$\max_{d_t, d_f} \tilde{C} = \frac{1}{MN} \sum_{m,n} \xi_{mn} \tilde{C}_n(m) \tag{15}$$

subject to

$$d_t \in \{1, \cdots, M\}, \tag{16}$$

$$d_f \in \{1, \cdots, N\}. \tag{17}$$

Since the number of possible solutions of (15) is MN, an optimal solution can be easily obtained by exhaustive search.

IV. Simulation Results

Consider an OFDM system for HSR which has one transmitter on a train and one receiver on railroad side. Assume that the variances of all channel coefficients are 1, the maximum Doppler frequency is $f_d = 100$ Hz, the maximum delay spread of the channel is $\sigma_t = 1\mu$s, and the noise variance is $N_0 = 1$. Suppose that the number of subchannels is $N = 12$, the number of time slots is $M = 7$, and the time slot duration is $T = 100$ ms.

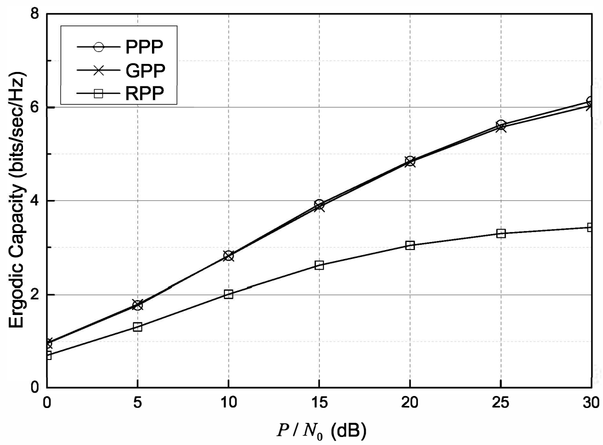

Fig. 2. Ergodic capacity for the proposed PPP and GPP schemes.

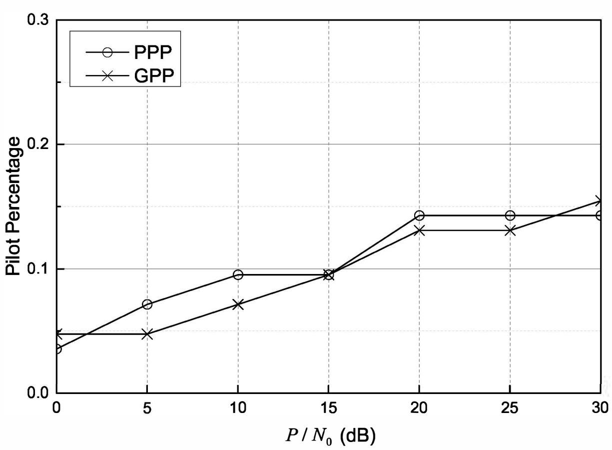

Fig. 3. Pilot percentage for the proposed PPP and GPP schemes.

Fig. 1 shows the ergodic capacity of a frame for the proposed PPP scheme. It is shown that the ergodic capacity for the PPP scheme increases as P/N_0 increases. It is also shown that the gap between the ergodic capacity and its lower bound increases as P/N_0 increases.

Fig. 2 shows the ergodic capacity for the proposed PPP and GPP schemes and, for comparison, random pilot placement (RPP) scheme. In the RPP scheme, the number of RBs is same as that of PPP scheme, while the locations of RBs are randomly selected among all blocks [15].

It is shown that the proposed PPP and GPP schemes achieve much higher ergodic capacity than that of the RPP scheme, although they use a loose lower bound, as shown in Fig. 1, to place pilots. It is also shown that the two proposed schemes achieve almost same ergodic capacity for all values of P/N_0.

Fig. 3 shows the pilot percentage for the proposed PPP and GPP schemes. It is shown that the pilot percentage for the two proposed schemes increases as P/N_0 increases. This result indicates that when the transmit power becomes large, using more pilot is effective to increase the ergodic capacity.

V. Conclusion

In this paper, we derive the ergodic capacity of an OFDM system for HSR. The lower bound of ergodic capacity is derived in closed form. We propose two pilot placement schemes, called greedy pilot placement (GPP) and periodic pilot placement (PPP) schemes, which maximize the lower bound of the ergodic capacity. It is shown that the proposed GPP and PPP schemes achieves much higher ergodic capacity than the random pilot placement scheme. It is also shown that the pilot percentage for the two proposed schemes increases as P/N_0 increases.

Appendix

Based on the Jensen's inequality, (5) is lower bounded by

$$\begin{aligned} C_n(m) &= E\left[\log_2\left(1+\frac{\gamma\left|\hat{h}_n(m)\right|^2}{\gamma|e_n(m)|^2+1}\right)\right] \\ &\geq E_{\left|\hat{h}_n(m)\right|^2}\left(\log_2\left(1+\frac{\gamma\left|\hat{h}_n(m)\right|^2}{\gamma(1-\rho_n(m)^2)+1}\right)\right) \\ &= \int_0^\infty \log_2\left(1+\frac{\gamma x}{\gamma(1-\rho_n(m)^2)+1}\right) f_{\left|\hat{h}_n(m)\right|^2}(x)dx \end{aligned} \tag{18}$$

where $f_{\left|\hat{h}_n(m)\right|^2}(x)$ is the pdf of the exponentially distributed random variable $\left|\hat{h}_n(m)\right|^2$ having mean $\rho_n(m)^2$, which is given by

$$p_{\left|\hat{h}_n(m)\right|^2}(x) = \frac{1}{\rho_n(m)^2}\exp\left(-\frac{x}{\rho_n(m)^2}\right). \tag{19}$$

Substituting (19) into (18), the lower bound of the ergodic capacity is given by

$$\begin{aligned} \tilde{C}_n(m) = \frac{1}{\rho_n(m)^2}\int_0^\infty \log_2\left(1+\frac{\gamma x}{\gamma(1-\rho_n(m)^2)+1}\right) \\ \cdot \exp\left(-\frac{x}{\rho_n(m)^2}\right)dx \end{aligned} \tag{20}$$

$$\tilde{C}_n(m) = \frac{\gamma(1-\rho_n(m)^2)+1}{\gamma\rho_n(m)^2}\exp\left(\frac{\gamma(1-\rho_n(m)^2)+1}{\gamma\rho_n(m)^2}\right)\int_1^{\infty}\log_2(v)\exp\left(-\frac{\left(\gamma(1-\rho_n(m)^2)+1\right)v}{\gamma\rho_n(m)^2}\right)dv$$
$$= A_n(m)\exp\left(A_n(m)\right)\int_1^{\infty}\log_2(v)\exp\left(-A_n(m)v\right)dv. \tag{22}$$

In order to calculate this integral term, we introduce a new integral variable as

$$v = 1 + \frac{\gamma x}{\gamma(1-\rho_n(m)^2)+1}. \tag{21}$$

Then, (18) is rewritten in (22) at the top of this page, where

$$A_n(m) = \frac{\gamma(1-\rho_n(m)^2)+1}{\gamma\rho_n(m)^2}. \tag{23}$$

Using integration by parts, (22) becomes

$$\tilde{C}_n(m) = \frac{1}{\ln 2}\exp\left(A_n(m)\right)\int_1^{\infty}\frac{1}{v}\exp\left(-A_n(m)v\right)dv. \tag{24}$$

By changing variable $u = A_n(m)v$, (24) is rewritten as

$$\tilde{C}_n(m) = \frac{1}{\ln 2}\exp\left(A_n(m)\right)\int_{A_n(m)}^{\infty}\frac{1}{u}\exp\left(-u\right)du. \tag{25}$$

By the definition of the incomplete Gamma function, we yields (6).

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